## Joint Seminar – CQSE, CTP, & CASTS

2019 Nov. 29, Friday

TIME Nov. 29, 2019, 2:30~3:30pm

TITLE Classical Ising model on 3D fractal lattice & Sierpiński carpet

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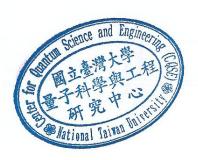
PLACE Rm716, CCMS & New Physics Building, NTU

## **Abstract**

The central interest of the condensed matter theory is that of understanding the phase transition and critical phenomena. Most of the effort so far has been focused on studying the systems with a regular structure, which possess the property of translational invariance. In the case of the translationally invariant systems, scale invariance occurs only at a critical point. As a consequence of such emergence of the scale invariance, the critical behavior is controlled only by global properties of the underlying structure, e.g., dimensionality and symmetries, which is a concept known as universality. Critical phenomena on fractal lattices are still relatively under-explored. Due to their non-integer dimensionality, fractal lattices lack the translational invariance; however, they possess a weaker type of symmetry, i.e., scale invariance. Scale invariance means that fractals exhibit self-similarity at every scale. Since the whole structure is self-similar at every scale, the influence of the more delicate details of the geometrical structure is amplified as the system size grows and thus the critical behavior becomes further governed by the additional geometric aspects such as ramification, lacunarity, connectivity, etc.

In my talk, I will focus on two systems: (1) Classical Ising model on a 3D fractal lattice and (2) Classical Ising model on the Sierpiński carpet.

--More details (Please see below)--



- (1) The current study is an extension of our earlier studies of the phase transition phenomena on a fractal lattice\*. Whereas the before-studied fractal lattice was embedded into the two-dimensional space, now, the embedding space is three dimensional. It can be expected, that such a change of the dimension would have a rather significant influence on the character of the phase transition of the system, such as the critical exponents. Moreover, there was no singularity found in the specific heat around the critical temperature in the case of the fractal with the Hausdorff dimension  $d^{(H)} = \ln 12 / \ln 4 \operatorname{prox} 1.792$ . The Hausdorff dimension of the currently-studied fractal lattice is exactly  $d^{(t)} = \ln 32 / \ln 4 = 2.5$ ; thus, singular behavior of the specific heat might be expected, as such behavior is already exhibited by a regular square-lattice Ising model.
- (2) One of the important questions which have not been considered in the literature so far is the dependence of the quantities such as order parameters on their specific location within the self-similar structure. Our current study focuses on the ferromagnetic Ising model on the Sierpiński carpet, for which several Monte Carlo studies in conjunction with the finite-size scaling method as well as Monte Carlo renormalization group have already been conducted. The difficulty of the study of fractal lattices by more traditional approaches such as the Monte Carlo methods can be illustrated by the fact that there are rather significant differences already in the estimates of the critical temperature \$T\_{\text{text}\{c\}}\$ among different authors (with some values closer to \$1.48\$ and others closer to \$1.50\$). Using the modified HOTRG method, we have confirmed the presence of the \$2^{\text{text}\{nd}\}\$ order phase transition, and we have determined the phase transition temperature to be \$T\_{\text{text}\{c\}}=1.4783\$. The magnetization exhibits a strong positional dependence with the critical exponent \$\beta\$ found to be different by a couple of orders of magnitude depending on the position of the imposed impurity tensors.
- \*J. Genzor, A. Gendiar, and T. Nishino, Phys. Rev. E {\bf 93} (2016) 012141.